The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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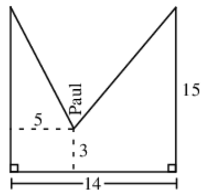
Topic Generator - Solution Set
Solutions

1. A “slackrope walker" is much like a tightrope walker except that the rope on which he performs is not pulled tight. Paul, a slackrope walker, has a rope tied to two15 m high poles which are 14 m apart. When he is standing on the rope 5 m away from one of the poles, he is 3 m above the ground. How long is the rope?



(A) 28 m (B) 30 m (C) 27 m (D) 26 m (E) 29 m

Source: 2005 Gauss Grade 8 #20

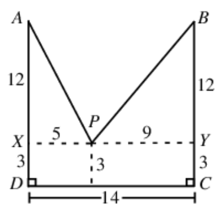
Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Perimeter | Pythagorean Theorem

Answer: A

Solution:

Label the points A, B, C and D, as shown.
Through P, draw a line parallel to DC as shown.
The points X and Y are where this line meets AD and BC.
From this, we see that $AX = YC = 15 - 3 = 12$.
Also, $PY = 14 - 5 = 9$.



To calculate the length of the rope, we need to calculate AP and BP, each of which is the hypotenuse of a right-angled triangle.
Now, $AP^2 = 12^2 + 5^2 = 169$ so $AP = 13$, and $BP^2 = 12^2 + 9^2 = 225$, so $BP = 15$.
Therefore, the required length of rope is $13 + 15$ or 28 m.

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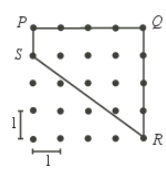
2. In the diagram, the horizontal distance between adjacent dots in the same row is 1. Also, the vertical distance between adjacent dots in the same column is 1. What is the perimeter of quadrilateral $PQRS$?



(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

Source: 2012 Pascal Grade 9 #14

Primary Topics: Geometry and Measurement

Secondary Topics: Perimeter | Quadrilaterals | Pythagorean Theorem

Answer: C

Solution:

The perimeter of quadrilateral $PQRS$ equals $PQ + QR + RS + SP$.
Since the dots are spaced 1 unit apart horizontally and vertically, then $PQ = 4$, $QR = 4$, and $PS = 1$.
Thus, the perimeter equals $4 + 4 + RS + 1$ which equals $RS + 9$.
We need to determine the length of RS.
If we draw a horizontal line from S to point T on QR, we create a right-angled triangle STR with $ST = 4$ and $TR = 3$.
By the Pythagorean Theorem, $RS^2 = ST^2 + TR^2 = 4^2 + 3^2 = 25$.
Since $RS > 0$, then $RS = \sqrt{25} = 5$.
Thus, the perimeter of quadrilateral $PQRS$ is $5 + 9 = 14$.

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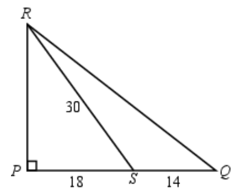
3. In $\triangle PQR$, $\angle RPQ = 90^\circ$ and S is on PQ. If $SQ = 14$, $SP = 18$, and $SR = 30$, then the area of $\triangle QRS$ is



(A) 84 (B) 168 (C) 210 (D) 336 (E) 384

Source: 2014 Pascal Grade 9 #15

Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Area | Pythagorean Theorem

Answer: B

Solution:

Solution 1
Since $\triangle RPS$ is right-angled at P, then by the Pythagorean Theorem, $PR^2 + PS^2 = RS^2$ or $PR^2 + 18^2 = 30^2$.
This gives $PR^2 = 30^2 - 18^2 = 900 - 324 = 576$, from which $PR = 24$, since $PR > 0$.
Since P, S and Q lie on a straight line and RP is perpendicular to this line, then RP is actually a height for $\triangle QRS$ corresponding to base SQ.
Thus, the area of $\triangle QRS$ is $\frac{1}{2}(24)(14) = 168$.

Solution 2
Since $\triangle RPS$ is right-angled at P, then by the Pythagorean Theorem, $PR^2 + PS^2 = RS^2$ or $PR^2 + 18^2 = 30^2$.
This gives $PR^2 = 30^2 - 18^2 = 900 - 324 = 576$, from which $PR = 24$, since $PR > 0$.
The area of $\triangle QRS$ equals the area of $\triangle RPQ$ minus the area of $\triangle RPS$.
Since $\triangle RPQ$ is right-angled at P, its area is $\frac{1}{2}(PR)(PQ) = \frac{1}{2}(24)(18+14) = 12(32) = 384$.
Since $\triangle RPS$ is right-angled at P, its area is $\frac{1}{2}(PR)(PS) = \frac{1}{2}(24)(18) = 12(18) = 216$.
Therefore, the area of $\triangle QRS$ is $384 - 216 = 168$.

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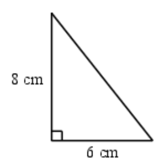
4. There is a square whose perimeter is the same as the perimeter of the triangle shown. The area of that square is



(A) 12.25 cm^2 (B) 196 cm^2 (C) 49 cm^2 (D) 36 cm^2 (E) 144 cm^2

Source: 2015 Gauss Grade 8 #16

Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Area | Perimeter | Pythagorean Theorem

Answer: D

Solution:

First, we must determine the perimeter of the given triangle.

Let the unknown side length measure x cm.

Since the triangle is a right-angled triangle, then by the Pythagorean Theorem we get $x^2 = 8^2 + 6^2$ or $x^2 = 64 + 36 = 100$ and so $x = \sqrt{100} = 10$ (since $x > 0$).

Therefore the perimeter of the triangle is $10 + 8 + 6 = 24$ cm and so the perimeter of the square is also 24 cm.

Since the 4 sides of the square are equal in length, then each measures $\frac{24}{4} = 6$ cm. Thus, the area of the square is $6 \times 6 = 36\text{ cm}^2$.

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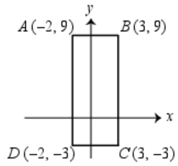
5. In the diagram, what is the length of BD?



(A) 13 (B) 17 (C) $\sqrt{205}$ (D) $\sqrt{160}$ (E) 15

Source: 2016 Gauss Grade 8 #20

Primary Topics: Geometry and Measurement

Secondary Topics: Coordinate Geometry | Quadrilaterals | Pythagorean Theorem

Answer: A

Solution:

Points D and C have equal y-coordinates, -3, and so side DC is parallel to the x-axis and has length $3-(-2)=5$.
Points B and C have equal x-coordinates, 3, and so side BC is parallel to the y-axis and has length $9-(-3)=12$.
That is, in $\triangle BCD$, sides DC and BC are perpendicular or $\angle BCD = 90^\circ$ with $BC = 9$ and $DC = 5$.
Using the Pythagorean Theorem, $BD^2 = DC^2 + BC^2 = 5^2 + 12^2$ and so $BD^2 = 25 + 144 = 169$ or $BD = \sqrt{169} = 13$ (since $BD > 0$).

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6. A line segment joins the points $P(-4,1)$ and $Q(1,-11)$. What is the length of PQ?

(A) 13 (B) 12 (C) 12.5 (D) 13.6 (E) 12.6

Source: 2019 Gauss Grade 8 #19

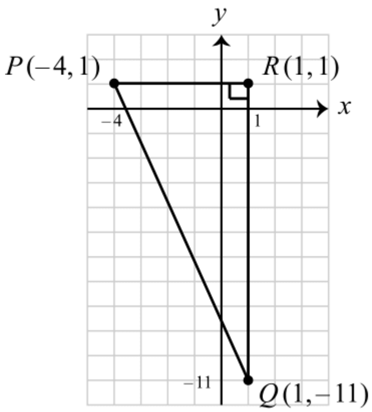
Primary Topics: Geometry and Measurement

Secondary Topics: Coordinate Geometry | Measurement | Pythagorean Theorem

Answer: A

Solution:

The horizontal line through P intersects the vertical line through Q at $R(1,1)$.



Joining P, Q, R creates right-angled $\triangle PQR$, with hypotenuse PQ.
The x-coordinate of P is -4 and the x-coordinate of R is 1, and so PR has length $1-(-4)=5$ (since P and R have equal y-coordinates).
The y-coordinate of Q is -11 and the y-coordinate of R is 1, and so QR has length $1-(-11)=12$ (since Q and R have equal x-coordinates).
Using the Pythagorean Theorem, $PQ^2 = PR^2 + QR^2$ or $PQ^2 = 5^2 + 12^2 = 25 + 144 = 169$, and so $PQ = \sqrt{169} = 13$ (since $PQ > 0$).
(Alternatively, we could have drawn the vertical line through P and the horizontal line through Q which meet at $(-4,-11)$.)

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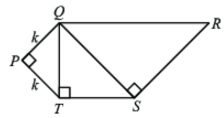
7. In the diagram, each of $\triangle QPT$, $\triangle QTS$ and $\triangle QSR$ is an isosceles, right-angled triangle, with $\angle QPT = \angle QTS = \angle QSR = 90^\circ$. The combined area of the three triangles is 56. If $QP = PT = k$, what is the value of k?



(A) $\sqrt{2}$ (B) 1 (C) 4 (D) 2 (E) $2\sqrt{2}$

Source: 2019 Pascal Grade 9 #18

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Triangles | Pythagorean Theorem

Answer: C

Solution:

By the Pythagorean Theorem,

$$QT^2 = QP^2 + PT^2 = k^2 + k^2 = 2k^2$$

Since $QT > 0$, then $QT = \sqrt{2}k$.
Since $\triangle QTS$ is isosceles, then $TS = QT = \sqrt{2}k$.
By the Pythagorean Theorem,

$$QS^2 = QT^2 + TS^2 = (\sqrt{2}k)^2 + (\sqrt{2}k)^2 = 2k^2 + 2k^2 = 4k^2$$

Since $QS > 0$, then $QS = 2k$.
Since $\triangle QSR$ is isosceles, then $SR = QS = 2k$.
Since $\triangle QPT$ is right-angled at P, its area is $\frac{1}{2}(QP)(PT) = \frac{1}{2}k^2$.
Since $\triangle QTS$ is right-angled at T, its area is $\frac{1}{2}(QT)(TS) = \frac{1}{2}(\sqrt{2}k)(\sqrt{2}k) = \frac{1}{2}(2k^2) = k^2$.
Since $\triangle QSR$ is right-angled at S, its area is $\frac{1}{2}(QS)(SR) = \frac{1}{2}(2k)(2k) = 2k^2$.
Since the sum of the three areas is 56, then $\frac{1}{2}k^2 + k^2 + 2k^2 = 56$ or $\frac{7}{2}k^2 = 56$ which gives $k^2 = 16$.
Since $k > 0$, then $k = 4$.

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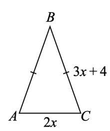
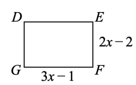
8. In the diagram, $\triangle ABC$ has $AB = BC = 3x + 4$ and $AC = 2x$ and rectangle $DEFG$ has $EF = 2x - 2$ and $FG = 3x - 1$.





The perimeter of $\triangle ABC$ is equal to the perimeter of rectangle $DEFG$. What is the area of $\triangle ABC$?

(A) 84 (B) 87.5 (C) 168 (D) 175 (E) 336

Source: 2024 Pascal Grade 9 #19

Primary Topics: Geometry and Measurement | Algebra and Equations

Secondary Topics: Triangles | Expressions | Perimeter | Area | Pythagorean Theorem

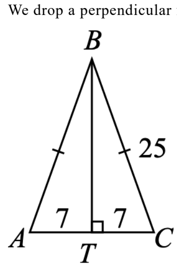
Answer: C

Solution:

The perimeter of $\triangle ABC$ is equal to $(3x + 4) + (3x + 4) + 2x = 8x + 8$.
The perimeter of rectangle $DEFG$ is equal to

$$2 \times (2x - 2) + 2 \times (3x - 1) = 4x - 4 + 6x - 2 = 10x - 6$$

Since these perimeters are equal, we have $10x - 6 = 8x + 8$ which gives $2x = 14$ and so $x = 7$.
Thus, $\triangle ABC$ has $AC = 2 \times 7 = 14$ and $AB = BC = 3 \times 7 + 425$.
We drop a perpendicular from B to T on AC.



Since $\triangle ABC$ is isosceles, then T is the midpoint of AC, which gives $AT = TC = 7$.
By the Pythagorean Theorem, $BT = \sqrt{BC^2 - TC^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24$.
Therefore, the area of $\triangle ABC$ is equal to $\frac{1}{2} \cdot AC \cdot BT = \frac{1}{2} \times 14 \times 24 = 168$.

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9. Equilateral triangle ABC has sides of length 4. The midpoint of BC is D, and the midpoint of AD is E. The value of EC^2 is
(A) 7 (B) 6 (C) 6.25 (D) 8 (E) 10

Source: 2022 Gauss Grade 8 #20

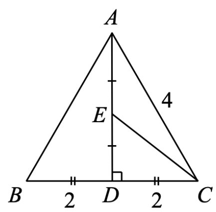
Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Expressions | Measurement | Pythagorean Theorem

Answer: A

Solution:

Since $\triangle ABC$ is equilateral and has sides of length 4, then $AB = BC = AC = 4$.
The midpoint of BC is D and so $BD = CD = 2$.
The midpoint of AD is E and so $AE = ED$.
Since $AB = AC$ and D is the midpoint of BC, then AD is perpendicular to BC, as shown.



Triangle ADC is a right-angled triangle, and so by the Pythagorean Theorem, we get
$(AC)^2 = (AD)^2 + (DC)^2$ or $4^2 = (AD)^2 + 2^2$, and so $(AD)^2 = 16 - 4 = 12$.
Similarly, $\triangle EDC$ is right-angled, and so by the Pythagorean Theorem, we get
$(EC)^2 = (ED)^2 + (DC)^2$ or $(EC)^2 = (ED)^2 + 2^2$.
Since $ED = \frac{1}{2}AD$, then $(ED)^2 = \frac{1}{2}AD \times \frac{1}{2}AD$ or $(ED)^2 = \frac{1}{4}(AD)^2$.
Since $AD^2 = 12$, then $(ED)^2 = \frac{1}{4} \times 12 = 3$.
Substituting, we get $(EC)^2 = 3 + 2^2$, and so $(EC)^2 = 7$.

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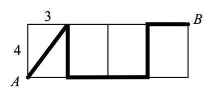
10. The diagram is made up of four congruent rectangles with dimensions 3 by 4.



The four rectangles are arranged side-by-side forming one larger rectangle. The left side of the large rectangle is labeled 4. The top (and bottom) is the total of four sides of length 3. A is the bottom left vertex of the large rectangle and B is the top right corner. A path is made up of line segments. It starts at A, moves diagonally up across the first rectangle, down the side shared by rectangles 1 and 2, across the bottoms of rectangles 2 and 3, up the shared side of rectangles 3 and 4, and the across the top of rectangle 4, ending at B.

What is the length of the path from A to B shown on the diagram?
(A) 22 (B) 21 (C) 19 (D) 20 (E) 23

Source: 2023 Gauss Grade 8 #11

Primary Topics: Geometry and Measurement

Secondary Topics: Quadrilaterals | Measurement | Pythagorean Theorem

Answer: A

Solution:

In the leftmost rectangle, the length of the path along the rectangle's diagonal, d, and the sides with lengths 3 and 4, form a right-angled triangle. Using the Pythagorean Theorem, we get $d^2 = 3^2 + 4^2$, and so $d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ (these are "3-4-5" right-angled triangles). The path from A to B includes one such diagonal, two vertical sides each of length 4, and three horizontal sides each of length 3, and thus has length $5 + (2 \times 4) + (3 \times 3) = 22$.

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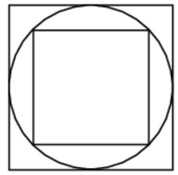
11. In the diagram, a circle is inscribed in a large square and a smaller square is inscribed in the circle. If the area of the large square is 36, the area of the smaller square is



(A) 15 (B) 12 (C) 9 (D) 24 (E) 18

Source: 2005 Gauss Grade 8 #21

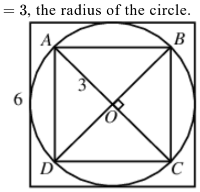
Primary Topics: Geometry and Measurement

Secondary Topics: Circles | Area | Quadrilaterals | Pythagorean Theorem

Answer: E

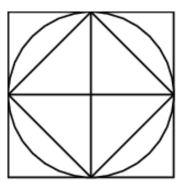
Solution:

Solution 1
Since the area of the large square is 36, then the side length of the large square is 6.
Therefore, the diameter of the circle must be 6 as well, and so its radius is 3.
Label the four vertices of the small square as A, B, C, and D.
Join A to C and B to D.
Since $ABCD$ is a square, then AC and BD are perpendicular, crossing at point O, which by symmetry is the centre of the circle.
Therefore, $AO = BO = CO = DO = 3$, the radius of the circle.



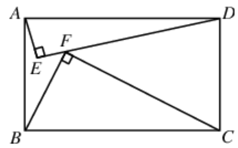
But square $ABCD$ is divided into four identical isosceles right-angled triangles.
The area of each of these triangles is $\frac{1}{2}bh = \frac{1}{2}(3)(3) = \frac{9}{2}$ so the area of the square is $4 \times \frac{9}{2} = 18$.

Solution 2
Rotate the smaller square so that its four corners are at the four points where the circle touches the large square.
Next, join the top and bottom points where the large square and circle touch, and join the left and right points.



By symmetry, these two lines divide the large square into four sections (each of which is square) of equal area.
But the original smaller square occupies exactly one-half of each of these four sections, since each edge of the smaller square is a diagonal of one of these sections.
Therefore, the area of the smaller square is exactly one-half of the area of the larger square, or 18.

12. In the diagram, right-angled triangles AED and BFC are constructed inside rectangle $ABCD$ so that F lies on DE. If $AE = 21$, $ED = 72$ and $BF = 45$, what is the length of AB?



(A) 50 (B) 48 (C) 52 (D) 54 (E) 56

Source: 2005 Pascal Grade 9 #25

Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Pythagorean Theorem

Answer: A

Solution:

By the Pythagorean Theorem in $\triangle AED$, $AD^2 = AE^2 + ED^2 = 21^2 + 72^2 = 5625$, so $AD = 75$.
Since $ABCD$ is a rectangle, $BC = AD = 75$. Also, by the Pythagorean Theorem in $\triangle BFC$,
$FC^2 = BC^2 - BF^2 = 75^2 - 45^2 = 3600$, so $FC = 60$.
Draw a line through F parallel to AB, meeting AD at X and BC at Y.
To determine the length of AB, we can find the lengths of FY and FX. Step 1: Calculate the length of FY
The easiest method to do this is to calculate the area of $\triangle BFC$ in two different ways.
We know that $\triangle BFC$ is right-angled at F, so its area is equal to $\frac{1}{2}(BF)(FC)$ or~$\frac{1}{2}(45)(60) = 1350$.
Also, we can think of FY as the height of $\triangle BFC$, so its area is equal to $\frac{1}{2}(FY)(BC)$ or $\frac{1}{2}(FY)(75)$.
Therefore, $\frac{1}{2}(FY)(75) = 1350$, so $FY = 36$.

(We could have also approached this by letting $FY = h$, $BY = x$ and so $YC = 75 - x$. We could have then used the Pythagorean Theorem twice in the two little triangles to create two equations in two unknowns.)

Since $FY = 36$, then by the Pythagorean Theorem,

$$BY^2 = BF^2 - FY^2 = 45^2 - 36^2 = 729$$

so $BY = 27$.
Thus, $YC = BC - BY = 48$.
Step 2: Calculate the length of FX
Method 1 -- Similar triangles
Since $\triangle AED$ and $\triangle FXD$ are right-angled at E and X respectively and share a common angle D, then they are similar.
Since $YC = 48$, then $XD = 48$.
Since $\triangle AED$ and $\triangle FXD$ are similar, then $\frac{FX}{XD} = \frac{AE}{ED}$ or $\frac{FX}{48} = \frac{21}{72}$ so $FX = 14$.

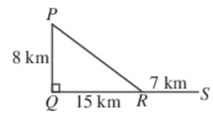
Method 2 -- Mimicking Step 1
Drop a perpendicular from E to AD, meeting AD at Z. We can use exactly the same argument from Step 1 to calculate that $EZ = \frac{504}{25}$ and that $ZD = \frac{1728}{25}$.
Since DF is a straight line, then the ratio $FX : EZ$ equals the ratio $DX : DZ$, ie.~$\frac{FX}{\frac{504}{25}} = \frac{48}{\frac{1728}{25}}$ or $\frac{FX}{504} = \frac{48}{1728}$ or $FX = 14$.

Method 3 -- Areas
Join A to F. Let $FX = x$ and $EF = a$. Then $FD = 72 - a$. Since $AE = 21$ and $ED = 72$, then the area of $\triangle AED$ is $\frac{1}{2}(21)(72) = 756$.
Now, the area of $\triangle AED$ is equal to the sum of the areas of $\triangle AEF$ and $\triangle AFD$, or

$$756 = \frac{1}{2}(21)(a) + \frac{1}{2}(75)(x)$$

so $21a + 75x = 1512$ or $a + \frac{25}{7}x = 72$.
Now in $\triangle FXD$, we have $FX = x$, $GD = 48$ and $FD = 72 - a$.
By the Pythagorean Theorem, $x^2 + 48^2 = (72 - a)^2 = (\frac{25}{7}x)^2 = \frac{625}{49}x^2$.
Therefore, $48^2 = \frac{576}{49}x^2$, or $x = 14$. Therefore, $AB = XY = FX + FY = 36 + 14 = 50$.

13. Asafa ran at a speed of 21 km/h from P to Q to R to S, as shown. Florence ran at a constant speed from P directly to R and then to S. They left P at the same time and arrived at S at the same time. How many minutes after Florence did Asafa arrive at point R?



(A) 0 (B) 8 (C) 6 (D) 7 (E) 5

Source: 2007 Pascal Grade 9 #22

Primary Topics: Geometry and Measurement

Secondary Topics: Triangles | Measurement | Rates | Pythagorean Theorem

Answer: E

Solution:

By the Pythagorean Theorem, $PR = \sqrt{QR^2 + PQ^2} = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$ km.
Asafa runs a total distance of $8 + 15 + 7 = 30$ km at 21 km/h in the same time that Florence runs a total distance of $17 + 7 = 24$ km.
Therefore, Asafa's speed is $\frac{30}{24} = \frac{5}{4}$ of Florence's speed, so Florence's speed is $\frac{4}{5} \times 21 = \frac{84}{5}$ km/h.

Asafa runs the last 7 km in $\frac{7}{21} = \frac{1}{3}$ hour, or 20 minutes.
Florence runs the last 7 km in $\frac{7}{\frac{84}{5}} = \frac{35}{84} = \frac{5}{12}$ hour, or 25 minutes.
Since Asafa and Florence arrive at S together, then Florence arrived at R 5 minutes before Asafa.

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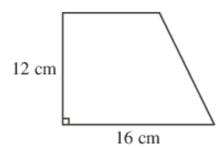
14. The trapezoid shown has a height of length 12 cm, a base of length 16 cm, and an area of 162 cm^2. The perimeter of the trapezoid is



(A) 51 cm (B) 52 cm (C) $49.\overline{6}$ cm (D) 50 cm (E) 56 cm

Source: 2011 Gauss Grade 8 #23

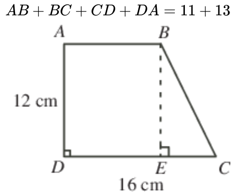
Primary Topics: Geometry and Measurement

Secondary Topics: Quadrilaterals | Area | Perimeter | Pythagorean Theorem

Answer: B

Solution:

We first label the trapezoid $ABCD$ as shown in the diagram below.
Since AD is the perpendicular height of the trapezoid, then AB and DC are parallel.
The area of the trapezoid is $\frac{AD}{2} \times (AB + DC)$ or $\frac{12}{2} \times (AB + 16)$ or $6 \times (AB + 16)$.
Since the area of the trapezoid is 162, then $6 \times (AB + 16) = 162$ and $AB + 16 = \frac{162}{6}$ or $AB + 16 = 27$, so $AB = 11$.
Construct a perpendicular from B to E on DC.
Since AB is parallel to DE and both AD and BE are perpendicular to DE, then $ABED$ is a rectangle.
Thus, $DE = AB = 11$, $BE = AD = 12$, and $EC = DC - DE = 16 - 11 = 5$.
Since $\angle BEC = 90^\circ$, then $\triangle BEC$ is a right-angled triangle.
Thus by the Pythagorean Theorem, $BC^2 = BE^2 + EC^2$ or $BC^2 = 12^2 + 5^2$ or $BC^2 = 169$ so $BC = 13$ (since $BC > 0$).
The perimeter of the trapezoid is $AB + BC + CD + DA = 11 + 13 + 16 + 12 = 52$ cm.



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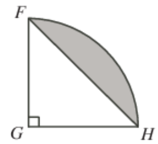
15. In right-angled, isosceles triangle FGH, $FH = \sqrt{8}$. Arc FH is part of the circumference of a circle with centre G and radius GH, as shown. The area of the shaded region is



(A) $\pi - 2$ (B) $4\pi - 2$ (C) $4\pi - \frac{1}{2}\sqrt{8}$ (D) $4\pi - 4$ (E) $\pi - \sqrt{8}$

Source: 2013 Gauss Grade 8 #22

Primary Topics: Geometry and Measurement

Secondary Topics: Circles | Area | Triangles | Pythagorean Theorem

Answer: A

Solution:

In $\triangle FGH$, $FG = GH = x$ since they are both radii of the same circle.
By the Pythagorean Theorem, $FH^2 = FG^2 + GH^2 = x^2 + x^2$, or $FH^2 = 2x^2$, and so $(\sqrt{8})^2 = 2x^2$ or $2x^2 = 8$ and $x^2 = 4$, so then $x = 2$ (since $x > 0$).
FG, GH and arc FH form a sector of a circle with centre G and radius GH.
Since $\angle FGH = 90^\circ$, which is $\frac{1}{4}$ of 360°, then the area of this sector is one quarter of the area of the circle with centre G and radius $GH = FG = 2$.
The shaded area is equal to the area of sector FGH minus the area of $\triangle FGH$.
The area of sector FGH is $\frac{1}{4}\pi(2)^2$ or $\frac{1}{4}\pi(4)$ or π.
The area of $\triangle FGH$ is $\frac{FG \times GH}{2}$ or $\frac{2\times 2}{2}$, so 2.
Therefore, the area of the shaded region is $\pi - 2$.

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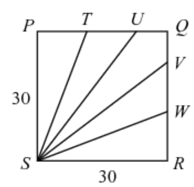
16. Square $PQRS$ has side length 30, as shown. The square is divided into 5 regions of equal area: $\triangle SPT$, $\triangle STU$, $\triangle SVW$, $\triangle SWR$, and quadrilateral $SUQV$. The value of $\dfrac{SU}{ST}$ is closest to



(A) 1.17 (B) 1.19 (C) 1.21 (D) 1.23 (E) 1.25

Source: 2018 Gauss Grade 8 #22

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Triangles | Pythagorean Theorem

Answer: B

Solution:

The area of square $PQRS$ is $(30)(30) = 900$.
Each of the 5 regions has equal area, and so the area of each region is $900 \div 5 = 180$.
The area of $\triangle SPT$ is equal to $\frac{1}{2}(PS)(PT) = \frac{1}{2}(30)(PT) = 15(PT)$.
The area of $\triangle SPT$ is 180, and so $15(PT) = 180$ or $PT = 180 \div 15 = 12$.
The area of $\triangle STU$ is 180.
Let the base of $\triangle STU$ be UT.
The height of $\triangle STU$ is equal to PS since PS is the perpendicular distance between base UT (extended) and the vertex S.
The area of $\triangle STU$ is equal to $\frac{1}{2}(PS)(UT) = \frac{1}{2}(30)(UT) = 15(UT)$.
The area of $\triangle STU$ is 180, and so $15(UT) = 180$ or $UT = 180 \div 15 = 12$.
In $\triangle SPT$, $\angle SPT = 90^\circ$. By the Pythagorean Theorem, $ST^2 = PS^2 + PT^2$ or $ST^2 = 30^2 + 12^2$ or $ST^2 = 900 + 144 = 1044$, and so $ST = \sqrt{1044}$ (since $ST > 0$).
In $\triangle SPU$, $\angle SPU = 90^\circ$ and $PU = PT + UT = 12 + 12 = 24$.
By the Pythagorean Theorem, $SU^2 = PS^2 + PU^2$ or $ST^2 = 30^2 + 24^2$ or $SU^2 = 900 + 576 = 1476$, and so $SU = \sqrt{1476}$ (since $SU > 0$).
Therefore, $\dfrac{SU}{ST} = \dfrac{\sqrt{1476}}{\sqrt{1044}}$ which is approximately equal to 1.189.
Of the answers given, $\dfrac{SU}{ST}$ is closest to 1.19.

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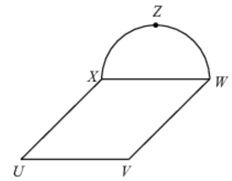
17. In the diagram, $UVWX$ is a rectangle that lies flat on a horizontal floor. A vertical semi-circular wall with diameter XW is constructed. Point Z is the highest point on this wall. If $UV = 20$ and $VW = 30$, the perimeter of $\triangle UVZ$ is closest to



(A) 95 (B) 86 (C) 102 (D) 83 (E) 92

Source: 2017 Pascal Grade 9 #22

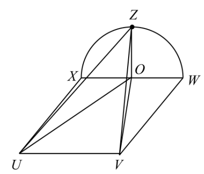
Primary Topics: Geometry and Measurement

Secondary Topics: Measurement | Perimeter | Pythagorean Theorem

Answer: B

Solution:

The perimeter of $\triangle UVZ$ equals $UV + UZ + VZ$.
We know that $UV = 20$. We need to calculate UZ and VZ.
Let O be the point on XW directly underneath Z.
Since Z is the highest point on the semi-circle and XW is the diameter, then O is the centre of the semi-circle.
We join UO, VO, UZ, and VZ.



Since $UVWX$ is a rectangle, then $XW = UV = 20$ and $UX = VW = 30$.
Since XW is a diameter of the semi-circle and O is the centre, then O is the midpoint of XW and so $XO = WO = 10$.
This means that the radius of the semi-circle is 10, and so $OZ = 10$ as well.
Now $\triangle UXO$ and $\triangle VWO$ are both right-angled, since $UVWX$ is a rectangle.
By the Pythagorean Theorem, $UO^2 = UX^2 + XO^2 = 30^2 + 10^2 = 900 + 100 = 1000$ and $VO^2 = VW^2 + WO^2 = 30^2 + 10^2 = 1000$.
Each of $\triangle UOZ$ and $\triangle VOZ$ is right-angled at O, since the semi-circle is vertical and the rectangle is horizontal.
Therefore, we can apply the Pythagorean Theorem again to obtain $UZ^2 = UO^2 + OZ^2$ and $VZ^2 = VO^2 + OZ^2$.
Since $UO^2 = VO^2 = 1000$, then $UZ^2 = VZ^2 = 1000 + 10^2 = 1100$ or $UZ = VZ = \sqrt{1100}$.
Therefore, the perimeter of $\triangle UVZ$ is $20 + 2\sqrt{1100} \approx 86.332$.
Of the given choices, this is closest to 86.

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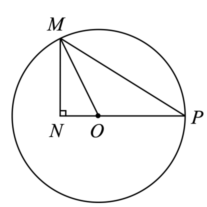
18. In the diagram, O is the centre of a circle with radius 87, and P and M lie on the circle. N is positioned inside the circle so that PN passes through O and is perpendicular to MN.



If $MN = 63$, what is the area of $\triangle PMN$?

(A) 3370.5 (B) 3496.5 (C) 4725.0 (D) 4630.5 (E) 4126.5

Source: 2023 Gauss Grade 8 #21

Primary Topics: Geometry and Measurement

Secondary Topics: Circles | Measurement | Area | Pythagorean Theorem

Answer: D

Solution:

Since OM is a radius of the circle, then $OM = 87$. $\triangle MNO$ is a right-angled triangle, and so by the Pythagorean Theorem, we get $OM^2 = MN^2 + NO^2$ or $87^2 = 63^2 + NO^2$, and so $NO^2 = 87^2 - 63^2 = 3600$. Since $NO > 0$, then $NO = \sqrt{3600} = 60$.
Since OP is also a radius, then $OP = 87$, and so $NP = NO + OP = 60 + 87 = 147$. The area of $\triangle PMN$ is equal to $\frac{1}{2} \times NP \times MN = \frac{1}{2} \times 147 \times 63 = 4630.5$.

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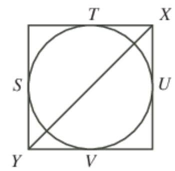
19. In the diagram, the circle is *inscribed* in the square. This means that the circle and the square share points S, T, U, and V, and the width of the square is exactly equal to the diameter of the circle. Rounded to the nearest tenth, what percentage of line segment XY is outside the circle?



(A) 29.3 (B) 28.3 (C) 33.3 (D) 25 (E) 16.7

Source: 2009 Gauss Grade 8 #23

Primary Topics: Geometry and Measurement

Secondary Topics: Circles | Percentages | Quadrilaterals | Pythagorean Theorem

Answer: A

Solution:

Since we are looking for the percentage of the whole length, we may take the side length of the square to be any convenient value, as the actual length will not affect the final answer.
Let us assume that the side length of the square is 2. Then the diameter of the circle is also 2 because the width of the square and the diameter of the circle are equal.
Using the Pythagorean Theorem, $XY^2 = 2^2 + 2^2 = 4 + 4 = 8$ or $XY = \sqrt{8}$.
The portion of line segment XY lying outside the circle has length XY minus the diameter of the circle, or $\sqrt{8} - 2$.
The percentage of line segment XY lying outside the circle is $\dfrac{\sqrt{8}-2}{\sqrt{8}} \times 100\% \approx 29.3\%$.

20. Points $A(-3,5)$, $B(0,7)$ and $C(r,t)$ lie along a line. If $BC = 4AB$ and $r > 0$, what is the value of $r+t$?

Source: 2025 Cayley Grade 10 #21

Primary Topics: Geometry and Measurement

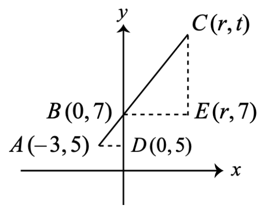
Secondary Topics: Coordinate Geometry | Equations Solving | Pythagorean Theorem

Answer: 27

Solution:

Solution 1:

Plot the points A, B, and C as well as $D(0,5)$ and $E(r,7)$ so that $\triangle ABD$ and $\triangle BCE$ have right angles at D and E, respectively.
These two right-angled triangles each have a horizontal side and a vertical side, as shown.



Since A, B, and C are all on the same line, AD is parallel to BE, and BD is parallel to CE, we must have that $\angle DAB = \angle EBC$ and $\angle DBA = \angle ECB$.
Hence, $\triangle ABD$ is similar to $\triangle BCE$.
Using common ratios, we get $\frac{BC}{AB} = \frac{EC}{DB} = \frac{BE}{AD}$.
Each of EC and DB is vertical, so their lengths are the difference between the y-coordinates of the two points.
Thus, $EC = t - 7$ and $DB = 7 - 5 = 2$.
Similarly, the lengths of BE and AD are each the different between the x-coordinates of the two points, giving $BE = r$ and $AD = 3$.
It is given that $BC = 4AB$, so $\frac{BC}{AB} = 4$. Therefore, $4 = \frac{EC}{DB} = \frac{t-7}{2}$ and $4 = \frac{BE}{AD} = \frac{r}{3}$.
Rearranging these two equations gives $8 = t - 7$ or $t = 15$ and $12 = r$, so $r + t = 27$.

Solution 2:

Using the distance formula,

$$AB = \sqrt{(7-5)^2 + (0-(-3))^2} = \sqrt{4+9} = \sqrt{13}$$

and

$$BC = \sqrt{(t-7)^2 + (r-0)^2} = \sqrt{(t-7)^2 + r^2}$$

It is given that $BC = 4AB$, so $4\sqrt{13} = \sqrt{(t-7)^2 + r^2}$. Squaring both sides gives $16 \times 13 = (t-7)^2 + r^2$.

It is also given that A, B, and C are on a common line. This implies that the slope of the segment AB is the same as the slope of the segment BC.
These slopes are $\frac{7-5}{0-(-3)} = \frac{2}{3}$ and $\frac{t-7}{r-0} = \frac{t-7}{r}$, respectively.
Setting the computed slopes equal, we have $\frac{t-7}{r} = \frac{2}{3}$, or $t - 7 = \frac{2}{3}r$.
Substituting into $16 \times 13 = (t-7)^2 + r^2$, we get the following equivalent equations.

$$\begin{aligned}
16 \times 13 &= \left(\frac{2}{3}r\right)^2 + r^2 \\
16 \times 13 &= \frac{4}{9}r^2 + r^2 \\
16 \times 13 &= \frac{13}{9}r^2 \\
16 \times 9 &= r^2 \\
\sqrt{16}\sqrt{9} &= \sqrt{r^2} \\
12 &= r
\end{aligned}$$

where the final equality is because r is assumed to be positive.

Therefore, we have $r = 12$, from which we get $t - 7 = \frac{2}{3} \times 12 = 8$, or $t = 15$.

The answer to the question is $r + t = 12 + 15 = 27$.